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Unsteady mass transfer in turbulent flow close to a rotating cylinder*

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Electrodiffusional methods of studying unsteady turbulent mass transfer involved measurement of a transient current characteristic $I(\tau)$ after step polarization of a rotating annular cylindrical 46 mm dia electrode at a fixed rotational velocity at $Re = (2\text{–}9) \times 10^4$ and $Sc = 2.4 \times 10^3$. The potassium ferri-ferrocyanide system with NaOH background electrolyte was used. An initial asymptote at $\tau \to 0$ served as a test. The similarity of the normalized transfer coefficient $K_+ = \beta/u_*$ with respect to the Reynolds number demonstrated turbulent flow development. Tests were aimed at determining the power n in the approximate law of attenuation of turbulent diffusion D_t in the y-direction normal to the wall $D_t/\nu = by_+^n$. A numerical solution of the unsteady turbulent diffusion equation obtained as a set of $\lg \Delta(\xi) = f(\xi)$ curves for $3 \leqslant n \leqslant 4$ with an interval 0.2, where $\Delta(\xi) = I(\xi)/I(\xi)_{\xi\to\infty} - 1$ has been achieved.

Notation

I diffusion current
C, C_0 and C_p concentration, concentration in the bulk liquid and polymer concentration, respectively
C_f drag of a Newtonian fluid
τ time
U linear velocity
ν kinematic viscosity
ω angular velocity
j flow
$y_+ = yu_*/\nu$, $\tau_+ = \tau u_*^2/\nu$ and $\bar{C} = (1 - C/C_0)$, dimensionless quantities

1. Introduction

A single rotating cylinder has been used in numerous studies of such problems as flow stability and generation of hydrodynamic noise [1] and laws of unsteady-state turbulent mass transfer of Newtonian and non-Newtonian fluids [2–6].

A fluid flow close to the surface of a single rotating cylinder quickly looses its stability and becomes turbulent. In the laminar region, the drag moment may be calculated by the formula [7]:

$$M = 4\pi uhr^2\,\omega \tag{1}$$

and in the turbulent region [8] by

$$1/\sqrt{(C_f)} = -0.6 + 4.07 \lg (Re\sqrt{C_f}) \tag{2}$$

where h is the cylinder height, $Re = \omega r^2/\nu$.

2. Theory

A rotating cylinder as a system for the study of turbulent mass transfer has an advantage over other systems. In the case of a solid electrode there is no entrance diffusion length, in contrast with tube and channel flows, as well as with flow around bodies. Mass transfer proceeds with a fully developed concentration profile since all radial derivatives of averaged characteristics are zero. The dimensionless parameter $K_+ = \beta/u_*$ characterizes turbulent mass transfer and is expressed as [9]

$$K_+ = \frac{n}{\pi}\sin\left(\frac{\pi}{n}\right) b^{1/n}\, Sc^{1/n-1} \tag{3}$$

Here b and n are coefficient and power exponent in the power law approximation of attenuation of turbulent diffusion, D_t, normal to the wall

$$D_t/\nu = by_+^n \tag{4}$$

where $y_+ = yu_*/\nu$ or u_* is the dynamic velocity and β the mass transfer coefficient.

Information about turbulent diffusion attenuation is contradictory. Of late, the value $n = 4$ is preferred. This also follows from the theory [10, 11] where the equations for velocity and concentration pulsations in a viscous sublayer are analysed. In [12], pulsational wall flow is approximated by monoharmonic pulsations. Bearing this in mind and using the maximum stability principle the following power law series follows:

$$D_t/\nu = 0.23 \times 10^{-5} y_+^3 + 18 \times 10^{-5} y_+^4 + \ldots \tag{5}$$

Note that contributions of the terms from Equation 5 are much different for small and large y_+. At $y_+ \geqslant 1$, the "fourth power law" prevails, whereas at $y \ll 1$,

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$D_t/\nu \sim y_+^3$. One or other model of the turbulent diffusion coefficient dependence on the distance to the wall can be chosen by comparing the appropriate model with experimental data. Models are based on the solution given by Equation 3 to the equation of steady-state turbulent mass transfer. For this, the experimental Schmidt number should be varied over a wide range, the condition of a developed turbulent flow being preserved at $Sc \geqslant 1$. Another model [13, 14] uses approximate solutions to the equation of unsteady-state mass transfer, and it is not necessary to change the Schmidt number for determining n. In this work, an attempt is made to increase the accuracy in determining n both by means of theoretical and experimental models. Such a theoretical model includes the numerical solution to the equation of turbulent mass transfer developing at a steady external flow velocity and stepwise variation of the wall concentration gradient.

For a rotating circular cylindrical electrode, the boundary-value problem with allowance for dimensionless variables is given by

$$\frac{\partial \bar{C}(y_+, \tau_+)}{\partial \tau_+} = \frac{\partial}{\partial y_+}\left(Sc^{-1} + \frac{D_t}{\nu}\right)\frac{\partial \bar{C}(y_+, \tau_+)}{\partial y_+}$$

$$\bar{C}(0, \tau_+) = 0; \quad \bar{C}(y_+, 0) = C_0; \quad \bar{C}(\infty, \tau_+) = \bar{C}_0, \tag{6}$$

where the turbulent diffusion coefficient is generally represented by the power law function $D_t/\nu = by_+^n$. For further analysis it is convenient to cast Equation 6 into a universal form by introducing the variables $\xi = (bSc)^{2/n}\tau_+/Sc$, $\eta = (bSc)^{1/n} y_+$. This gives the standard equation:

$$\frac{\partial \bar{C}(\eta, \xi)}{\partial \xi} = \frac{\partial}{\partial \eta}\left[(1 + \eta^n)\frac{\partial \bar{C}}{\partial \eta}\right]$$

$$\bar{C}(\eta, 0) = 1; \quad \bar{C}(0, \xi) = 0; \quad \bar{C}(\theta, \xi) = 1. \tag{7}$$

Numerical solution was obtained using the developed implicit scheme. The numerical computational program was tested by comparing the homogeneous solution to the molecular diffusion problem ($\tau \to 0$) with the known analytical solution.

As a result, a family of $\lg \Delta(\xi) = f(\xi)$ curves was obtained for different values $3 \leqslant n \leqslant 4$ with an interval 0.2, where $\Delta(\xi) = j(0, \xi)/j(0, \infty) - 1$.

3. Experimental details

3.1. *Methods*

Tests were made on two rotating single cylinders. One cylinder was 166 mm in dia; a cylindrical 1000 mm dia. tank contained the electrolyte solution. A solid nickel ring 5.2 mm high and a round 0.4 mm dia. platinum electrode were set flush with the cylinder surface. The other cell consisted of a rotating single 46 mm dia. cylinder immersed in a square 350 × 350 mm tank filled with electrolyte solution. To measure mass transfer, nickel rings 1 mm and 0.332 mm high, were set on the outer cylinder surface. In both cases, the rotational speed was smoothly regulated and registered using an optic/electronic system and frequency meter. The signal from the probes mounted on the rotating cylinders entered via the mercury current collector, the analog/digital converter, and finally it was inputed to a PC to produce tabular experimental data and diagrams. The electrolyte solution was composed of potassium ferri-ferrocyanide (2.5×10^{-2} kmol m^{-3}) and NaOH (2 kmol m^{-3}). The diffusion current was measured to determine drag and to derive $C_f = f(Re)$ where $Re = \omega r^2/\nu$ and r is the cylinder radius.

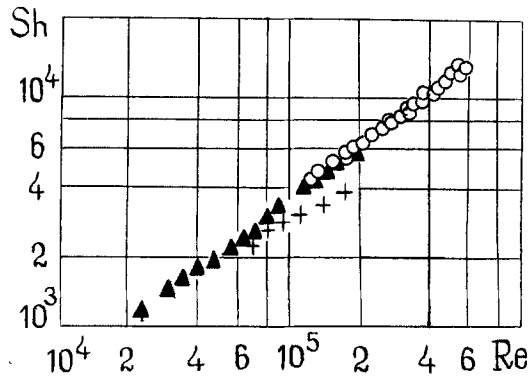

Fig. 1. Turbulent mass transfer from a rotating cylinder, diameter, d: (○) 166, (▲) 46, (+) 46 mm. Data for added PAA solution (C_p = 500 p.p.m.)

3.2. *Results and discussion*

The measured steady-state turbulent mass transfer (coefficients) are shown in Fig. 1 and are approximated by

$$Sh = kRe^{0.8} \tag{8}$$

General statements of the theory of steady-state turbulent diffusion with a developed wall concentration profile imply

$$K = \text{const} \times Sc^{1/n} \tag{9}$$

Experimental determination of the power in the law of turbulent diffusion attenuation involves analysis of the transient diffusion current characteristic upon step polarization of a rotating circular electrode within the range $Re = 4 \times 10^4$–2.5×10^5. This meant that the experimental $I_{exp} = f(\tau)$ data were compared with the theoretical $I_T = f(\tau)$ ones, thus yielding values of n in Equation 4. The molecular diffusion coefficient, D, was determined by the total transient characteristic for the limiting case $\tau \to 0$. Experimental data were interpreted as $\lg \Delta(\xi) = f(\xi)$. Comparison with the numerical solution in Equation 7 is given in Fig. 2. Each of the experimental points was obtained by generalizing the data of three to five transient diffusion current characteristics. Selecting an averaging procedure is bound up with specific transient turbulent mass transfer properties. For diffusion current pulsations it is desirable to average over several data records, for example, at $\tau = 0.05$ s; 0.07 s; 0.09 s and so on. From the conditions for Equation 7, the most

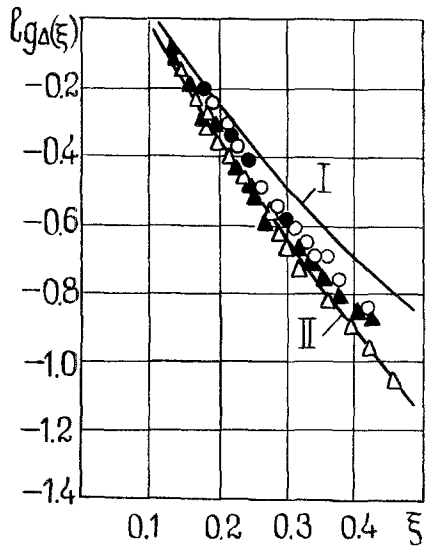

Fig. 2. Transient diffusion current characteristic lg $\Delta(\xi)$ against ξ for electrolyte solution. (○, ●) interpretation of experimental data at $n = 3$ for $Re = 1.16 \times 10^5$ and 1.6×10^6, respectively; (△, ▲) the same at $n = 4$; I, II, theory for $n = 3$ and $n = 4$, respectively.

appropriate curve for approximating the experimental data was taken within the range $0.1 < \xi < 0.5$. Analysis of all the characteristics of diffusional current allows the mechanism of turbulent diffusion at different distances from a wall to be revealed. As the parameter ξ – a dimensionless time of unsteady mass transfer development – increases the diffusion boundary layer grows. In reaching a steady-state the layer undergoes different stages of interaction with turbulent vortices.

The lowest deviation of the experimental results from the calculated curve in Fig. 2 suggests that both $n = 3$ and $n = 4$ are possible over the narrow range $0.1 < \xi < 0.2$. Over a wider range, $0.2 < \xi < 0.4$, n tends to 4. Using $n = 4$ and the measured steady-state mass transfer coefficients, as well as allowing for Equations 8 and 9, the semi-empirical formula for mass transfer at a rotating cylinder at $Re > 4 \times 10^4$ for a Newtonian fluid may be given as

$$Sh = 0.051\, Re^{0.8}\, Sc^{0.25}. \qquad (10)$$

The available data on steady-state mass transfer at rotating cylinders, reviewed in detail in [3], were basically obtained by the method of dissolution of solid bensoic acid. Our data at $Re > 2 \times 10^4$ agree well. The experimental data interpreted as $K_+ = f(Re)$ are given in Fig. 3 and demonstrate that at $Re > 8 \times 10^4$, K_f does depend on Re, thus characterizing steady-state turbulent flow. Using $K_+ = 4.3 \times 10^4$ and Equation 3, yields $b = 8.3 \times 10^4$. Upon simple transformation Equation 3 becomes

$$Sh = 0.152 \sqrt{\frac{C_f}{2}}\, Re\, Sc^{1/4}. \qquad (11)$$

Equation 11, showing the analogy between mass and momentum transfer, may be applied to determine the rotating cylinder surface drag by measuring mass transfer from a rotating circular electrode.

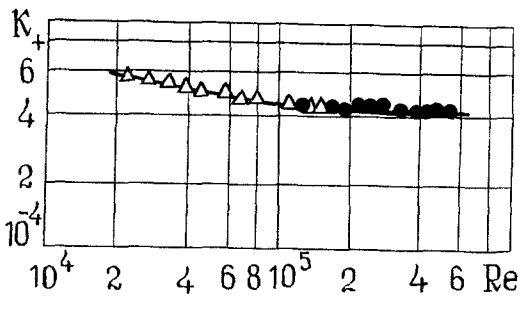

Fig. 3. Parameter K_+ against the Reynolds number. Cylinder diameter, d: (●) 166 and (△) 46 mm.

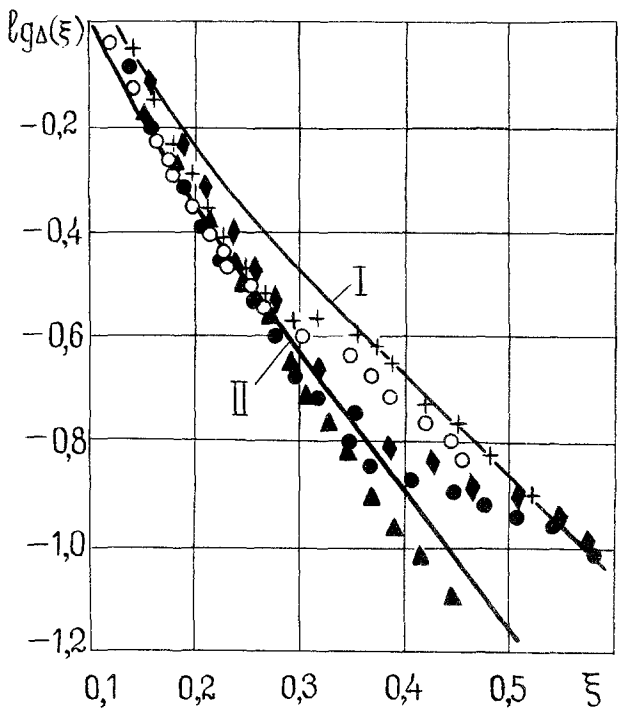

Fig. 4. Transient diffusion current characteristic lg $\Delta(\xi)$ against ξ for PAA solution at $C_p = 500$ p.p.m. (+, ◆) interpretation of experimental data at $n = 3$ for $Re = 1.16 \times 10^5$ and 1.6×10^5, respectively; (○, ●) the same at $n = 4$; (▲) data for degradated PAA solution. I, II, theory for $n = 3$ and $n = 4$, respectively. As the experimental data for 1 mm and 0.332 mm rings did not show scatter, special notation was not introduced.

It is interesting to estimate the effect of drag-reducing polymer solutions on turbulent diffusion. In tests, Separan-AP-273 polyacrilamide (PAA) solutions at concentrations C_p 500 and 1000 p.p.m. were used. Data on the transient diffusion current characteristic of two circular electrodes 1 and 0.332 mm high are presented in Fig. 4. In solutions which display Tom's effect three zones may be distinguished: (i) $0.1 < \xi < 0.2$ where $n = 3$ and $n = 4$ are equally probable; (ii) $0.2 < \xi < 0.3$, n tends to 4; and (iii) $0.3 < \xi < 0.5$, n tends to 3. These zones are most distinct at lower rotation speeds ($Re = 1.16 \times 10^5$). At higher rotation speeds the rate of polymer solution degradation increases. The ability of the solution to reduce the drag was tested on a coaxially-cylindrical rotating system with direct measurement of the torque of one of the cylinders. For the PAA solutions, not exhibiting Tom's effect due to degradation (experimental points 5), the transient characteristic behaves like that of water without PAA admixtures. It should be noted that region (i) requires more detailed study.

In measuring steady-state mass transfer the difference between "fresh" and degradated PAA solutions is also observed. Thus, the influence of drag-reducing PAA admixtures on n manifests itself in region (iii). It is still difficult to explain this fact. It is possibly due to the dominating polymer influence on the transient external turbulent boundary layer, as compared to the viscous sublayer.

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